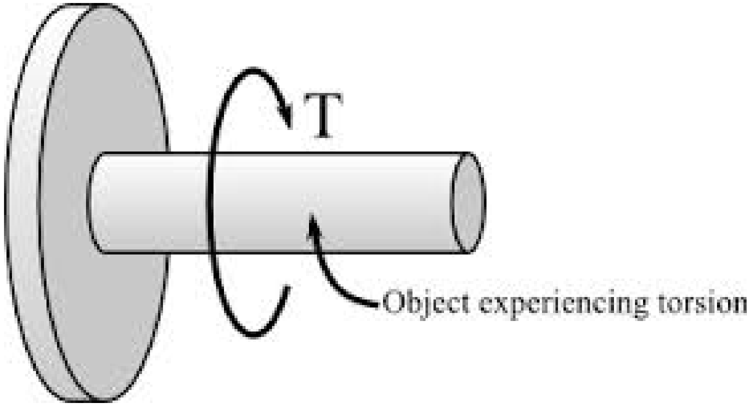
TORSION

5

Torsion 181



5.1 Torsional Deformation of a Circular Shaft

- *Torque* is a moment that tends to twist a member about its longitudinal axis.
- Its effect is of primary concern in the design of drive shafts used in vehicles and machinery.

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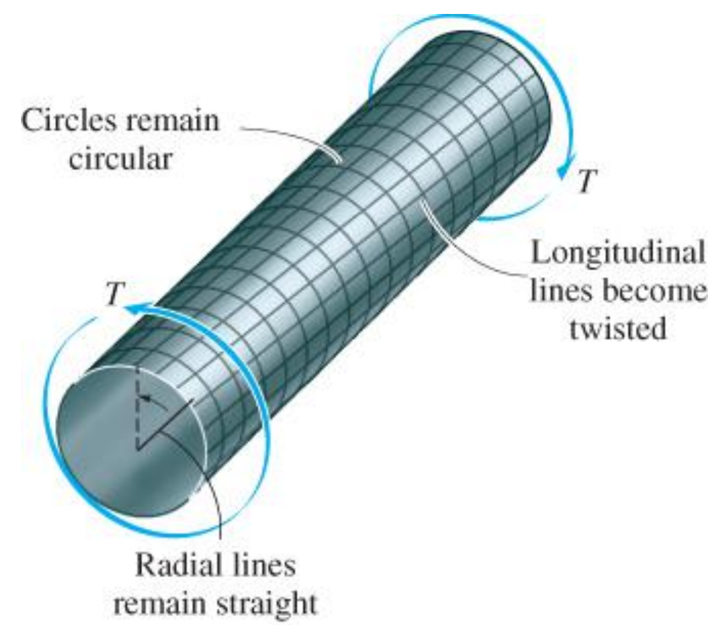
- Twisting causes the circles to *remain circles* , and each longitudinal grid line deforms into a helix that intersects the circles at equal angles.

- Also, the cross sections at the ends of the shaft will remain *flat,* and radial lines *remain straight* during the deformation.

- We can assume that if the angle of twist is *small* , the *length of the shaft* and its *radius* will *remain unchanged.*

Before deformation
(a)



After deformation
(b)

5.2 The Torsion Formula

When an external torque is applied to a shaft, it creates a corresponding internal torque within the shaft.

In this section, we will develop an equation that relates this internal torque to the shear stress distribution on the cross section of a circular shaft or tube.

If the material is linear-elastic, then Hooke's law applies, $\tau = G\gamma$, and leads to a corresponding ***linear variation in shear stress*** along any radial line on the cross section.

τ will vary from zero at the shaft's longitudinal axis to a maximum value, τ_{max} , at its outer surface.

This variation is shown in Fig. 5–5 on the front faces of a selected number of elements, located at an intermediate radial position $\boldsymbol{\rho}$ and at the outer radius *c.*

Due to the proportionality of triangles, we can write

$$\boxed{\tau = \left(\frac{\rho}{c}\right)\tau_{\max}}$$

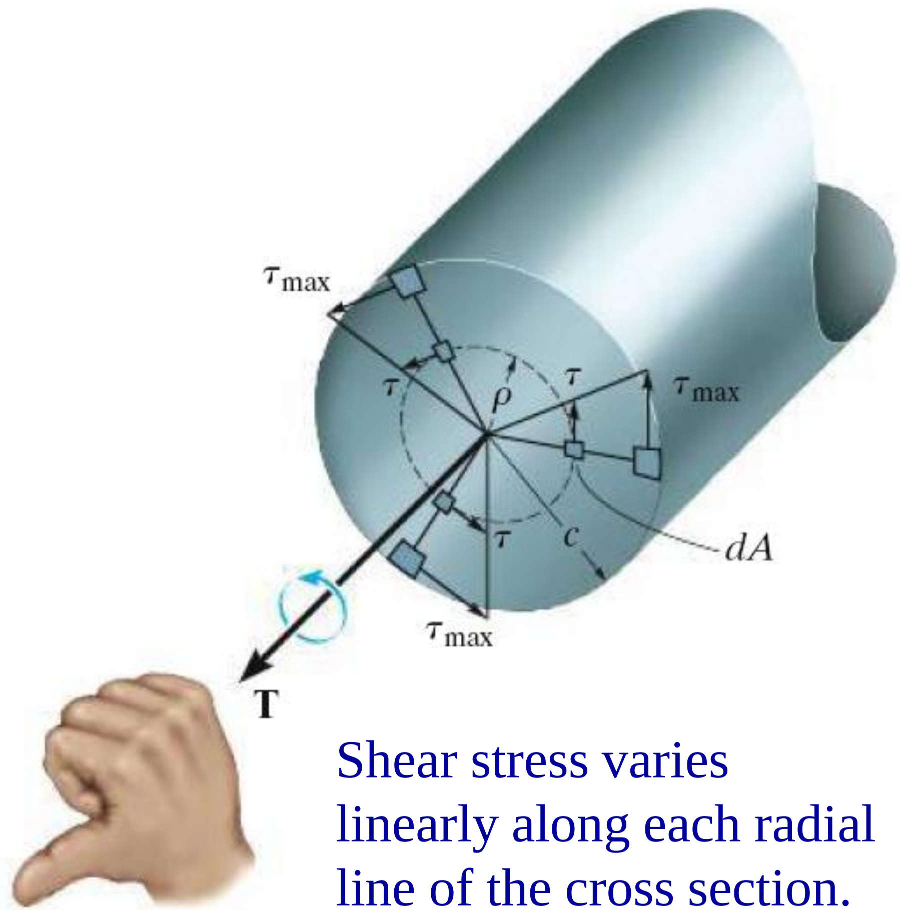


Fig. 5–5

Shear stress varies linearly along each radial line of the cross section.

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Specifically, each element of area dA, located at ρ, is subjected to a force of $dF = \tau\, dA$. The torque produced by this force is $dT = \rho(\tau\, dA)$. We therefore have for the entire cross section

$$T = \int_A \rho(\tau\, dA) = \int_A \rho\left(\frac{\rho}{c}\right)\tau_{\max}\, dA \qquad (5\text{–}4)$$

Since $\tau_{\max}/c$ is constant,

$$T = \frac{\tau_{\max}}{c}\int_A \rho^2\, dA \qquad (5\text{–}5)$$

- The integral in the equation can be represented as the polar moment of inertia ***J***, of shaft's x-sectional area computed about its longitudinal axis

$$\tau_{max} = \frac{Tc}{J}$$

τ_{max} = max. shear stress in shaft, at the outer surface.

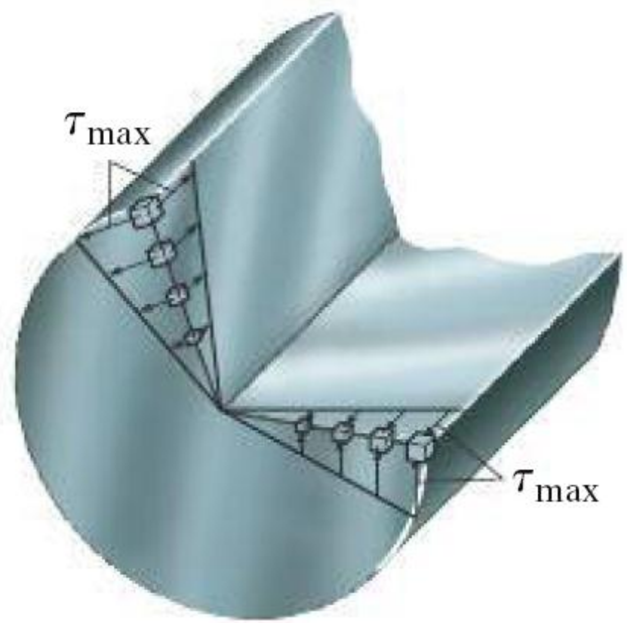
T = resultant internal torque acting at x-section, from method of sections & equation of moment equilibrium applied about longitudinal axis.

J = polar moment of inertia at x-sectional area

C = outer radius of the shaft

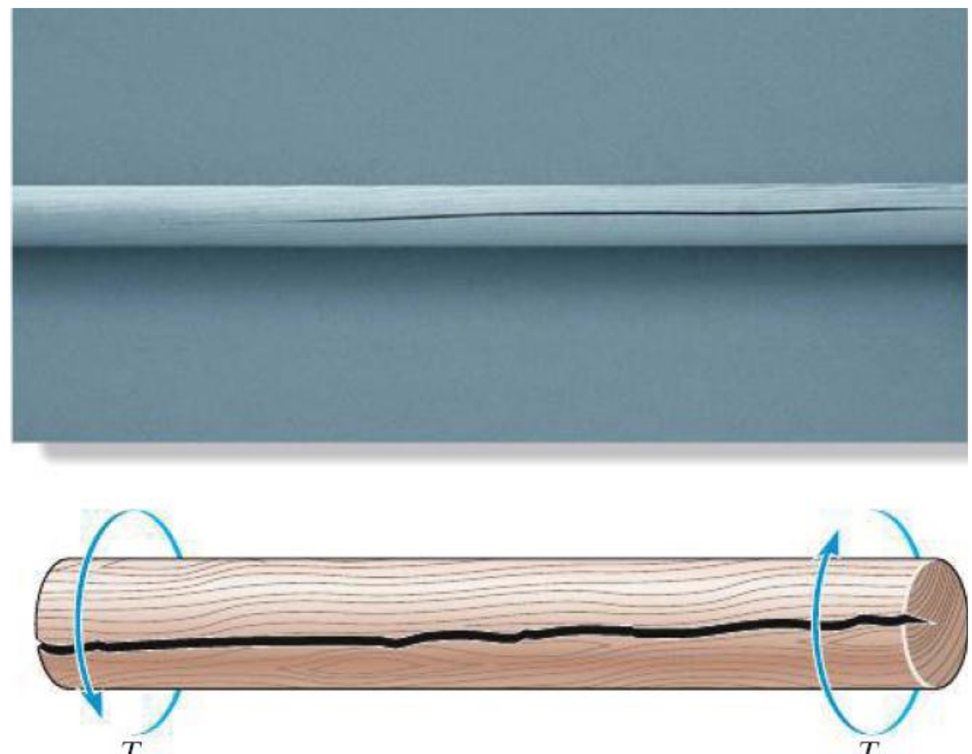
Not only does the internal torque *T* develop a linear distribution of shear stress along each radial line in the plane of the cross-sectional area.

But also an associated shear-stress distribution is developed *along an axial plane* , Fig. 5–7 b



Shear stress varies linearly along each radial line of the cross section.
(b)

shafts made from wood tend to *split* along the axial plane when subjected to excessive torque, Fig. 5–8 . This is because wood is an anisotropic material.



Failure of a wooden shaft due to torsion.

Fig. 5–8

TORSION

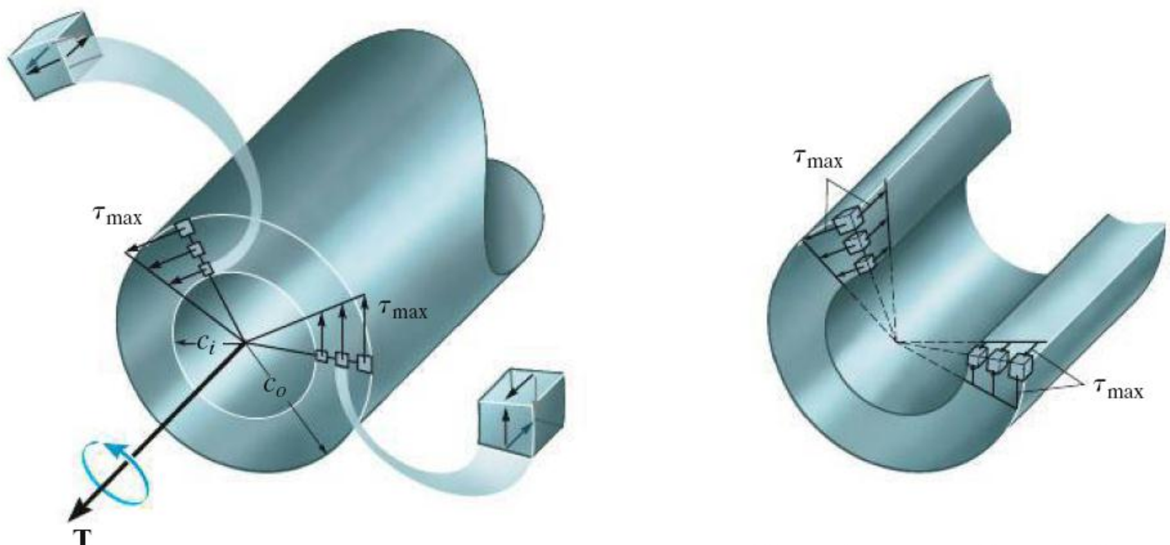
❖ Tubular Shaft.

If a shaft has a tubular cross section, with inner radius C_i and outer radius C_0

then from Eq. 5–8 we can determine its polar moment of inertia by subtracting J for a shaft of radius $\boldsymbol{C_i}$ from that determined for a shaft of radius $\boldsymbol{C_0}$. The result is

$$J = \frac{\pi}{2}\left(c_o^4 - c_i^4\right)$$

(5–9)



Shear stress varies linearly along each radial line of the cross section.

(a) (b)

Fig. 5–9

TORSION

The tubular drive shaft for this truck was subjected to an excessive torque resulting in failure caused by yielding of the material.

Engineers deliberately design drive shafts to fail before torsional damage can occur to parts of the engine or transmission.

❖ Absolute maximum torsional stress

- Need to find location where ratio Tc / J is a maximum.
- Draw a torque diagram (internal torque T vs. x along shaft)
- Sign Convention: T is positive, by right-hand rule, is directed outward from the shaft
- Once internal torque throughout shaft is determined, maximum ratio of Tc / J can be identified

Procedure for analysis

❖ Internal loading

- Section shaft perpendicular to its axis at point where shear stress is to be determined
- Use free-body diagram and equations of equilibrium to obtain internal torque at section.

❖ Section property

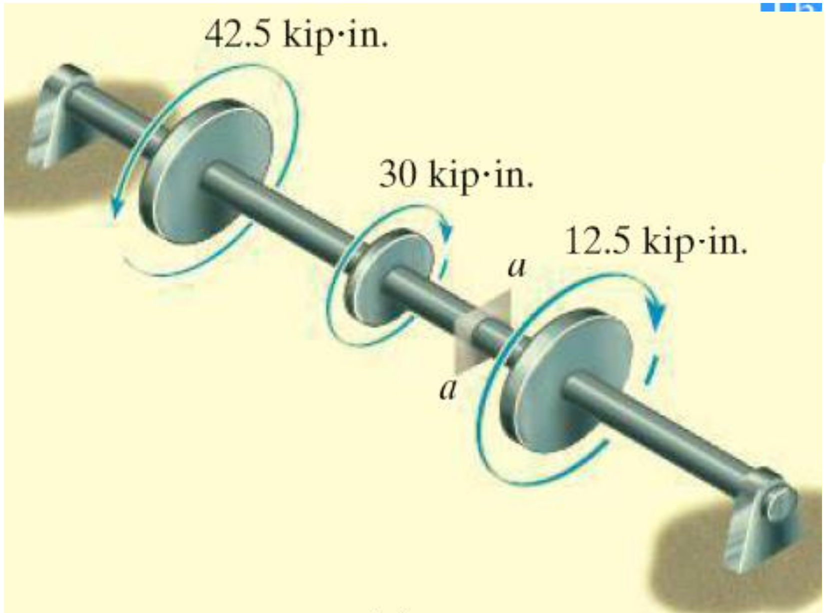
- Compute polar moment of inertia and x-sectional area
- For solid section, $J = \pi c^4/2$
- For tube, $J = \pi(c_o^4 - c_i^2)/2$

Strength of Materials: Second Class

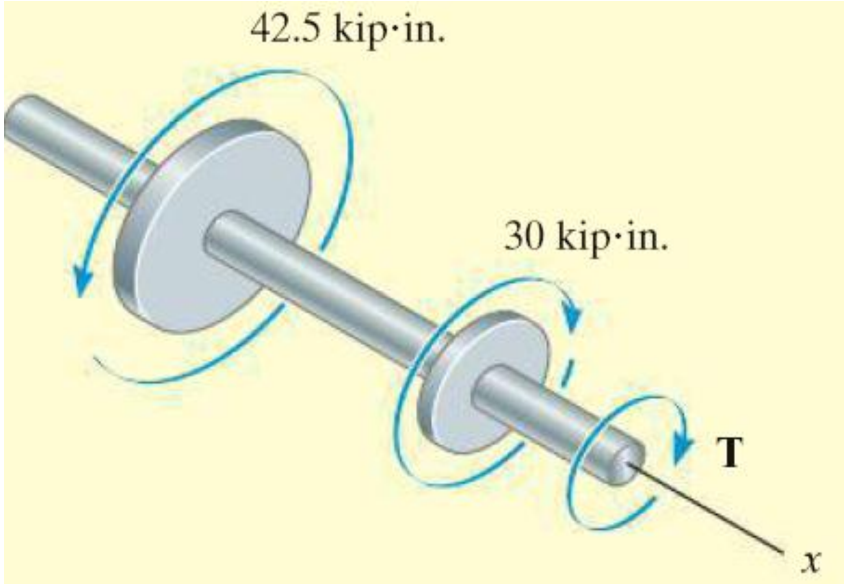
- Shear stress
- Specify radial distance ρ, measured from center of x-section to point where shear stress is to be found
- Apply torsion formula, $\tau = T\rho / J$ or $\tau_{\max} = Tc / J$
- Shear stress acts on x-section in direction that is always perpendicular to ρ

Strength of Materials: Second Class

EX1:- The shaft shown in Fig. 5–11 a is supported by two bearings and is subjected to three torques. Determine the shear stress developed at points $\boldsymbol{A}$ and $\boldsymbol{B}$, located at section $\boldsymbol{a-a}$ of the shaft, Fig. 5–11 c.



(a)



(b)

SOLUTION

Internal Torque.

The internal torque at section *a–a* will be determined from the free-body diagram of the left segment, Fig. 5–11*b*. We have

$$\Sigma M_x = 0; \quad 42.5 \text{ kip} \cdot \text{in.} - 30 \text{ kip} \cdot \text{in.} - T = 0 \quad T = 12.5 \text{ kip} \cdot \text{in.}$$

Section Property. The polar moment of inertia for the sha

$$J = \frac{\pi}{2}(0.75 \text{ in.})^4 = 0.497 \text{ in.}^4$$

Shear Stress. Since point A is at $\rho = c = 0.75$ in.,

$$\tau_A = \frac{Tc}{J} = \frac{(12.5 \text{ kip} \cdot \text{in.})(0.75 \text{ in.})}{(0.497 \text{ in.}^4)} = 18.9 \text{ ksi}$$

Likewise for point B, at $\rho = 0.15$ in., we have

$$\tau_B = \frac{T\rho}{J} = \frac{(12.5\ \text{kip}\cdot\text{in.})(0.15\ \text{in.})}{(0.497\ \text{in.}^4)} = 3.77\ \text{ksi}$$

Directions of the stresses on elements A and B established from direction of resultant internal torque **T**.

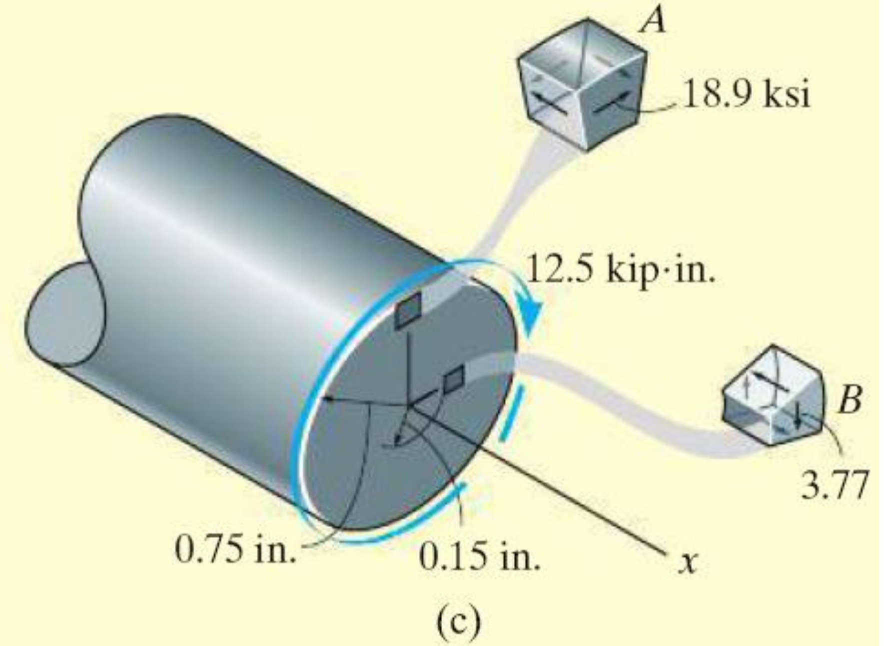


Fig. 5–11